DON’T TO MATH IN THE COCKPIT

Notes in this column will be used to either interpret a concept, provide a “second opinion,” or just offer a humorous example.

THE "M" WORD

The physical laws of the universe react precisely the same every time. Once we understand their process and relationship we can predict how they will react to help us or do us harm. The bad news is that mathematics is very helpful in describing relationships in all aspects of science. The good news is that we are not going to memorize a bunch of formulas or do a bunch of math exercises. We assume you already know all the math you need. We will explain what we need as we go.

There are several simple mathematical concepts we use in flying airplanes. Luckily these operations can all be done on the ground. From this point on, this should be our mathematical mantra -

DON'T DO MATH IN THE COCKPIT.

This means that we either have to do all our calculations ahead of time, find some job-performance-aids, or find some simple rules of thumb. Before we taxi out we must be sure we know:

1. which direction we should go?
2. how long it will take to get there?
3. how much fuel do we need?
4. do we have enough runway at both ends of the flight?
5. will we be within the weight and balance limits for the entire flight?

We can use an E6-B air navigation computer to make these calculations. The E6-B is a hand-held “whiz wheel” developed in the early 1900’s. There are also several electronic equivalents available. During the flight, we can use either of these to help with changes we experience on the flight. In many cases, simple Rules Of Thumb (ROT) can be applied to provide Ball Park Estimates (BPE). If time allows, we should be as accurate as necessary. In flight we should be able to find a usable estimate.

MEASUREMENTS

We use several different units to measure the same things. We measure airspeed in knots (kts), or nautical miles per hour. We gave up on mph in the 1960’s, so some old aircraft still show them. We measure altitude in feet. Some places in Europe use meters.

Round Off Error: when combining numbers in any mathematical action we can be accurate only to the lowest number of decimal places in any of the terms.

If we then added 2.5 to this product we would have to knock of another decimal place because the 2.5 could really be anywhere between 2.4500000 and 2.5444444.

Measure it with a micrometer, mark it with a crayon and cut it with an axe. We must be practical as to how the mathematical information is to be used.

How accurately do we have to measure things? When measuring things, we need to realize that we can only assume accuracy to the last significant digit. if we are multiplying the following numbers, we have to consider that they are only accurate to the minimum number of decimal places of any number. For example:

$$\begin{array}{r} 3.281 \\ \underline{\times \;\; 1.15} \\ 3.77315 \end{array}$$

3.281 could actually be 3.2805000 to 3.2814999999 rounded off for convenience. 1.15 may also be some wild number rounded off to only two decimal points. Therefore this product, 3.77315 may only be accurately expressed as: 3.77

If we are trying to find the circumference of a cylinder with a tolerance of .001 inch we don't need to carry the value of π out to more than four decimal places. If on the other hand, we are trying to fire a rocket to intercept the orbit of Jupiter and land exactly on the big black spot, we need to stretch our figures out several more decimal places.

For the Knowledge Test however, it is important, to keep at least all decimal places in the calculation until the very end. For real world problems, we may be able to be much less accurate.

USABLE AVIATION MATH

There are several requirements for mathematical computations. However they are all really quite simple to understand. Let's start with navigation. We must be able to measure our course line both in direction and distance. We measure the direction from True North with a protractor. We will discuss this in more detail when we discuss navigation.

We measure our distance in nautical miles (nm) because our airspeed indicators are calibrated in knots (kts) - or nautical miles per hour.

We use indicated airspeed (IAS) to measure the aerodynamic effects on the airplane. To be absolutely accurate we use calibrated airspeed (CAS) which takes out installation errors. We will have to use math to find our true airspeed (TAS). This will be used to determine the time required to cover the distance.

These operations will be made easier by the use of a flight oriented calculator. The generic E-6B, or the electronic equivalent will make these calculations much easier.

DIRECTION OF FLIGHT

My friend was a B-52 Navigator in Viet Nam trying to thread the needle between two heavily defended sites. This interchange actually took place.

Nav: "Turn left one-degree."
Pilot: "Don't be silly! Nobody can turn just one-degree!"
Nav: "OK, turn right 359-degrees."

Pilots fly a specific compass heading. In order to find it we must convert from True - to Magnetic - to Compass headings. True headings are related to the axis of the earth. The only way we can accurately align with true north is to point to the North Star at night. Fortunately for daytime navigation, most of the country was surveyed with reference to true north. You can align with the major north-south roads or farm fence lines. This doesn't work in Ohio, Louisiana or a few other locations that were surveyed relative to river banks.

Unfortunately, the magnetic north pole is related to a large spinning mass of ferrous metal in the earth's core. At present our compasses point to northern Canada. As we will see later, the location of the magnetic north and south pole is slowly, but continuously moving at the rate of about 30 miles per year. This causes local variations to change about 0.1° per year. We can correct for this by updating our navigation databases and charts. We can use known lines of "variation" to correct for each location. These lines are shown on all current navigation charts.

First math requirement: add or subtract the variation.

This magnetic variation in the US northwest is currently (2015) at roughly 16° E. This means that if we face the North Star, the magnetic north pole is 16° to the east - or to the right of the star. To convert from true to magnetic, we subtract that easterly variation. **"East is Least" when going from True to Magnetic.**

FOR	N	030	060	E	120	150
STEER	001	030	062	091	119	150
FOR	S	210	240	W	300	330
STEER	184	214	242	270	299	328

Figure 1: Compass Deviation Card

Second math requirement: add or subtract the error.

In addition, there are minor compass errors due to installation and magnetic fields around the cockpit which require a compass deviation card. This is displayed on or near the magnetic compass. In most cases this error is minor. It only makes a practical difference in three cases - ADF navigation, long distance navigation without navaids, or on the FAA Knowledge Test. To convert the compass heading to the required magnetic heading we look for our compass heading under the "STEER" column, and set our heading indicator to the magnetic heading indicated in the "FOR" column. For instance, referring to Figure 1 To the left, if your magnetic compass indicated 328 you should set your heading indicator to 330. The gyro heading indicator will remain consistent for turns to any direction until precession takes over.

ROT for converting Indicated Airspeed (IAS) to True Airspeed (TAS): TAS increases approximately 2% per 1000 ft

True airspeed (TAS) is corrected for altitude and temperature from indicated airspeed (IAS). All of this is can be done with the E-6B or electronic calculator. As a Rule of Thumb, TAS increases over IAS by roughly 2% per 1000 feet. At 10,000 feet, an Indicated airspeed of 100 kts would be approximately 120 kts True.

Wind

When we take off we are at the mercy of the movement of the air mass we inhabit. Unfortunately we do not fly where we point the airplane. If the air is moving from the south at 20 kts, then we must consider that as one of the vectors that make up our flight path.

Any direction and velocity can be expressed as a vector. If is made up of multiple factors we can find the sum of vectors. Airplane travel is the sum of flight path and wind path vectors.

Vectors?

A vector has both direction and speed. We can indicate a vector by a line in the appropriate direction. If we are flying west - 270° at 100 kts, and the wind is blowing us from the south - 180° at 20 kts, then our actual travel will be the result of drawing each line as follows:

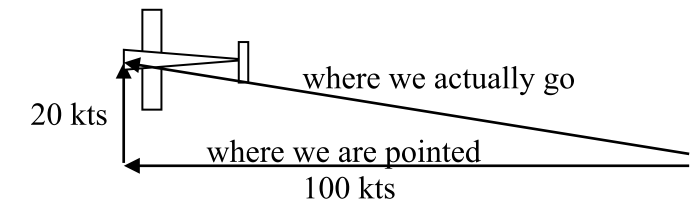


Figure 2: Drift

As a rule of thumb, a direct crosswind with a velocity of 10% of your true airspeed will cause a drift of 6°. Each additional 10% will cause an additional 6° of drift. This is true up to 60% of your true airspeed, then it starts to get trigonometrically wild. If the wind is above that, land! You are not going anywhere.

ROT
XWind of 10% of TAS = 6° Drift.

10% = 6°
20% = 12°
30% = 18°
40% = 24°
50% = 30°
60% = 36°

We of course want to fly along our intended course, so we calculate the wind triangle using either a protractor or E6-B computer to measure the angular difference (in this case approximately 12°) and turn toward the wind.

For instance if we wanted to go on a course of 270° we would draw our wind triangle thus:

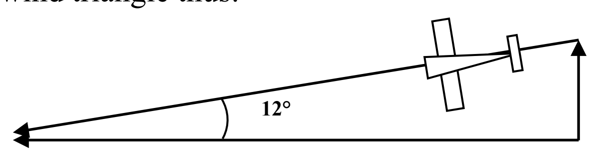


Figure 3: Drift correction

and fly a heading of 258°. With this correction we will stay on our desired westerly course. Because this causes us to go slightly farther, our ground speed will be less than TAS.

How fast are we going across the ground? In actual flight we need to know if our planning was correct. We check our groundspeed by marking our time at two points separated by a known distance. No in-cockpit math is required. Use the E6-B, but the formula is:

Distance A to B / minutes flown * 60

GROUND SPEED:
For Example,
A to B is 42 nm.
It took 24 min.
42 / 24 * 60 = 105 kts GS

It is important to know how to correct for wind. Your instructor will point out easier ways than this. Your flight calculator will lead you through easy steps. More on this when we get to navigation.

"The only time you have too much fuel is when you are on fire." (Anon , 1926)

TAKEOFF DISTANCE

Altitude	-10°C	0°C	10°C	20°C	30°C
Sea Level	820	930	1045	1170	1295
2000 ft	980	1045	1195	1320	1430
4000 ft	1195	1340	1450	1595	1710
6000 ft	2100	2320	2550	2810	3150

Figure 4: Example of Takeoff Distance Chart from Pilot's Operating Handbook

They will ask for the distance at 3600 ft and 15°C. First interpolate 3000 for both 10°C and 20°C, then interpolate the value for 15°C

1. **Find the ratio** of the distance above the lowest altitude to the selected altitude.

2. **Find the difference** in takeoff distances between 4000 ft and 2000 ft. at both 20°C and 15°C. Multiply by the ratio, and add it to the lower altitude takeoff distance.

3. **Interpolate** these two values for the difference in temperature.

Keep all the decimals until you are forced to choose an answer. Never choose an answer that is longer than your calculation.

Real World ROT for runway required: Choose the worst-case values from the chart and add a comfort factor for old engines and dirty wings.

FUEL CONSUMPTION

We calculate our approximate fuel consumption in gallons per hour (GPH). We can only be reasonably accurate if we fly with a constant power setting. Our miles-per-***gallon*** will change drastically with headwinds or tailwinds.

It is mathematically possible to fly 100 nm for three hours while burning 9-GPH. We would go 300 miles on 27 gallons. If our fuel tank holds 27 gallons, the last few miles will be fraught with anxiety - and maybe an unplanned adventure. A few more knots of headwind could cause you to "almost make it." Most pilots like to have an extra hour's worth after landing.

RUNWAY LENGTH

What is the shortest runway you will land on today? If we intend to re-use the airplane it is best to compute takeoff distances first. These are usually longer, and increase more dramatically with altitude and temperature. Luckily the manufacturer provides tables or charts with calculated best-case figures for takeoff and landing. It is important to remember that the manufacturer produced these performance values with a brand new airplane and a test pilot.

For the FAA knowledge test you will be asked to interpolate between altitudes and temperatures.

For example, if you were asked the minimum runway length at an altitude of 3600 feet at 15°C from the table on the left, you would be expected to interpolate from between the 2000 and 4000 runway requirement at 15°C.

15°C - 10°C = 5
20°C - 10°C = 10   5 / 10 = 0.5

At 2000msl, [1320 - 1195 =125 * 0.5 = 62.5ft] [1320 - 62.5 = 1257.5]
At 4000msl, [1595 -1450 = 145 * 0.5 = 72.5ft] [1595 - 72.5 = 1522.5]

Then find the distance at 3600 ft. 3600 is 1600 feet more than 2000. The difference between 2000 and 4000 is 2000 feet. The difference at each temperature would be 0.8 times the difference between the two values.

3600 - 2000 = 1600
4000 - 2000 = 2000   then 1600 / 2000 = 0.8

1522.5 - 1257.5 = 265 * 0.8 = 212

At 3600 msl at 15°C, takeoff distance is 1257.5 + 212 = **1469.5ft**

For the test, don't choose an answer less than 1470ft. 1469 won't make it.

There is a FAR requiring all trees around airports to stop growing when they reach 50-feet. Trees do not obey this regulation. There's a relatively short Fir tree in my yard that is 75 ft. tall. There are many taller ones in the forest.

In the real world no math is required. Just use the worst-case values from the various adjacent columns. In the above problem the worst-case would be 1595 feet. You may wish to add a "comfort factor" to avoid the panic as you approach the trees. Most pilots add 1000 feet, or 50% of the value. Remember - these charts will allow you to clear the 50-foot obstacle by zero feet.

WEIGHT AND BALANCE

Required design G-loads for various certification categories:

Normal Category: 3.8 G
Utility Category: 4.4 G
Acrobatic Category: 6.0 G

Each aircraft has weight limitations related to strength of structure, and performance. Each element of the structure is designed to sustain acceleration forces (G-loads) of 3.8 to 4.4 times its allowable gross weight. Beyond that, rivets may pop, or something bend. Catastrophic failure will not occur until 1.5 times those G-loads. At maneuvering Speed (V_A) the airplane will stall at the design load limit.

The location of a weight on a teeter totter can cause greater effect if farther from the fulcrum. This distance is expressed as an **Arm**.

The effect of the weight at that distance is called the **Moment**. In the simplest terms:

Weight X Arm = Moment.
(lbs) (inches) (in*lb)

We can find the balance point by dividing the sum of the pound/inches (Moment) by the total pounds (weight) to find the inches (arm, or CG location).

Moment / Weight = Arm
(in*lb) / (lbs) = (inches)

For convenience, the **Datum** from which the **Arm** is measured is usually moved to a point near the front of the aircraft.

If you are overweight you are a test pilot. Takeoff, landing and cruise performance charts are based on an aircraft at the maximum gross weight. If you are heavier than that, you cannot extrapolate to estimate your performance. At some percentage over your weight limit, you will have to taxi to your destination.

The balance point must be within established limits - fore and aft to provide controllability and stability. To calculate the Center of Gravity (CG) we multiply the ***weight*** **(lbs)** of each item of load by its ***arm (in)*** to find the **moment (lb/in)**. Then you must divide the total of all the moments by the total of all the weights. This will provide the actual location of the **CG.** Most aircraft manufacturers provide a simple method for calculating this. More on this when we study Weight and Balance.

You must also calculate your Zero Fuel weight and balance to see how much fuel you can carry. You might find that you have to leave one of kids home. You will also find out if you will be within balance limits if you have to use all your fuel.

If you are out of balance either forward or aft, you may have problems of control, stability, or stall/spin recovery. More on this as you study Aerodynamics.

USFUL TRIGONOMETRIC CONCEPTS:

Phythagorean Theorem: "The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides." (Pythagoras used this term because it wasn't being used for anything else.)

Pythagoras was Right

Pythagoras discovered a relationship between the angles and lengths of **right triangles** - in which one of the angles is 90°. First we have to make sense of the definitions of the useful relationships between all angles and lengths. We are most concerned with angles, their sine and tangent. Figure 5 below shows a right triangle. It also shows that any triangle can be broken down into two right triangles.

For convenience, the right angle will be labeled as "C" and the other two angles will be labeled "A" and "B." The sides opposite each angle will be labeled in small letters "a," "b" and "c." The longest line - between the two non-right angles is called the hypotenuse. **The hypotenuse (c) is always the longest side.**

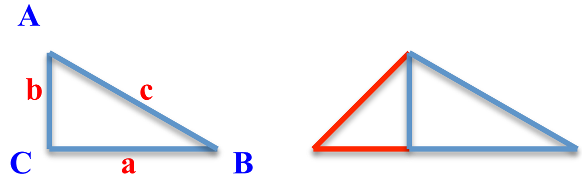


Figure 5: All triangles contain right triangles

The secret is in the sine. The **sine** is simply the relationship between the length of the side opposite either non-right angle, and the hypotenuse. This can be used in many ways. The **cosine** is the ratio between the adjacent side and the hypotenuse. The cosine of either non-right angle is the sine of the other.

The 60:1 Rule How wide is one-degree? We can find the length of side-b by the inverse of sine-B. The sine of one-degree is 0.07145. The inversion of sine (1°) in 57.299. Let's round that up to 60. If we measure the distance across 1-degree at 60 nm, it will be one-nm across. **Coincidence** - a nautical mile is very nearly 6076.12 ft. OK, 6080 for short. OK 6000 ft for convenience.

When flying down a standard 3-degree glide-slope on the ILS, at what distance would we expect to intercept it when flying 3000 ft above the surface? For each nautical mile from the runway, one-degree elevation would be 100 ft. Three-degrees would be 300 ft. The glide-slope would be 300 higher for each nm. Therefore, 10 nm from the transmitter the GS will be 3000 ft above the surface.

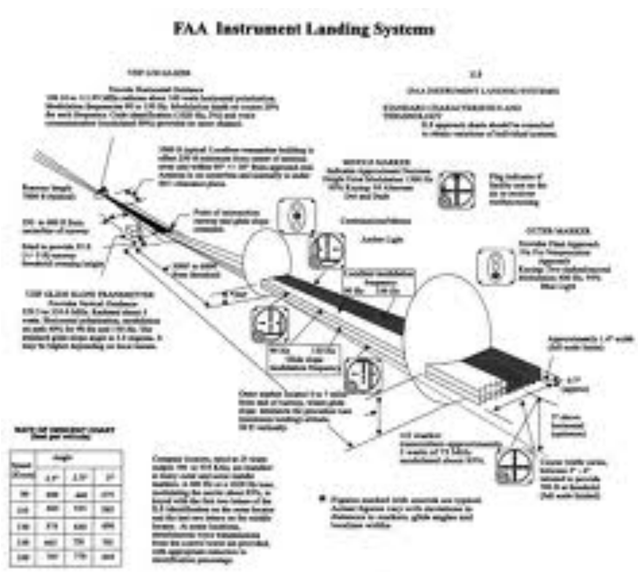


Figure 6: Technical diagram of the ILS system from the Aeronautical Information Manual.

The side opposite the 90° angle is called the **hypotenuse.** Because of the nature of right triangles, this has to be the longest side.

$a^2 + b^2 = c^2$

Where:
A, **B** and **C** are angles
a, **b** and **c** are sides

Sine is the ratio between two legs
Sine of angle A = the opposite leg divided by the hypotenuse
Sine A = a/c

All triangles all have 180°s worth of angles. If one angle is 90°, the other two have to equal 90°.

The **cosine** is simply the sine of the other acute angle of the triangle.
Cosine (A) = Sine (B).

The sign over the door at the USAF Air Warfare Command Instrument Instructor School says simply,

60:1

This ratio has so many uses in aviation. One-degree is one-mile wide at 60-miles.

Sine (1°) = 0.07145

$\frac{1}{0.07145}$ = 57.299 (OK, **60**)

6000 ft / 60 = 100 ft

The ILS glide-slope is normally three-degrees above horizontal.

Luckily for our ROT below,
100 fpm x 60 sec = 6000 ft (1 kt).

This 60:1 Rule also helps figure out our required vertical speed for various ground speeds along the ILS course. If we are traveling 60 kts we are covering 6,000 feet per feet per minute across the ground. During the same time we have to descend 300 feet. At 120 kts we travel 12,000 feet per minute. During this minute we have to descend 600 feet. The ratio holds for any groundspeed. At 100 kts groundspeed, descend at 500 fpm.

60 kts = Glide-slope = 300 fpm.
100 kts = Glide-slope = 500 fpm
120 kts = Glide-slope = 600 fpm.

ROT for GLIDESLOPE
VS = 1/2 GS x 10

Is there a rule-of-thumb here? The required vertical speed (fpm) to stay on the glide slope is simply 1/2 of the groundspeed (GS) in kts times 10. To stay on the glide-slope at 100 kts you must descend at 500 fpm. The tables provided on instrument approach charts vary slightly from this because ***we*** rounded up and down liberally. However, the Rule Of Thumb is plenty accurate.

ROT for PITCH CONTROL:
A change of one-degree of pitch = 100 ft per min (Not from math, from experience. This works on any airplane.)

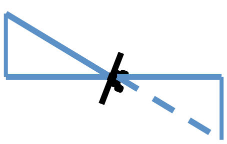
Wind Triangle: As we saw in our first ROT regarding crosswind drift correction, a crosswind is equal to **10% of our true airspeed in kts will cause 6-degrees of drift.** Using the 60:1 rule, at 60 kts, a 6-kt crosswind is equal to 6-degrees. At 100 kts a 10 kt crosswind will cause 6-degrees of drift.

At 420 kts a 12° drift shows that you are flying with an 84 kt crosswind component.

Bank Angle vs. G-load: As we will see in aerodynamics, when you bank an aircraft to turn, part of the lift generated by the wing is vectored to the side. The total vertical component of lift must still match the weight of the aircraft. The G-load is represented by the ration of the length of the hypotenuse to one-G – the length of the vertical line. The G-load is represented by the inverse of the cosine of the bank angle. The G-load in a 60-degree bank would be 2-times the weight of everything in the aircraft - including you. Using this function you can find the G-load for any bank angle. There is also a very convenient graph in your text.

The G-load is the inverse of the cosine of the bank angle.

$$\frac{1}{\text{cosine }(60)} = 2$$



There is a whole lot more to trigonometry that might be valuable to engineers, but knowing these simple concepts help you understand.

Stall-Related Speeds vs. Load Factor
All stall-related speeds increase or decrease as a function of the **square root of the load factor**. This can be the G-load, or the fraction of maximum gross weight at which you are loaded. The new stall-related speed is a function of the square root of the load factor times the stall-related speed. If you bank 60-degrees in an airplane that stalls at 55 kts, your new stall speed (Vs_1) is 55 kts * $\sqrt{2G}$) or (55 * 1.4) You might stall at 78 kts.

$$\mathbf{Vs_1 = Vs * \sqrt{\text{Load Factor}}}$$

For gross weights less than maximum:

$$\frac{\text{Today's weight } 2000}{\text{Max Gross } 2500} = 0.8 \text{ or } 80\%$$

$\sqrt{0.8}$ = roughly 0.9

Quite often the manufacturer will only publish performance speeds at the maximum allowable gross weight. You can calculate today's stall related-speeds based on the percentage of today's gross weight / maximum gross weight. This works for all stall related speeds including takeoff and climb (Vr, Vx Vy), maneuvering speed (Va) and landing speed. If your aircraft lists these speeds for different weights, you can validate this concept.

Some handy rough $\sqrt{\text{BPE}}$s:
If your load is:
60% $\sqrt{0.6}$ = 0.75
70% $\sqrt{0.7}$ = 0.85
80% $\sqrt{0.8}$ = 0.90
90% $\sqrt{0.9}$ = 0.95

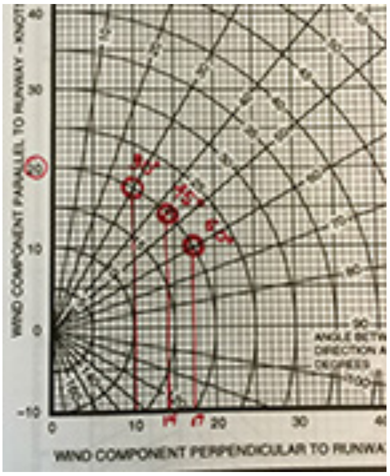


Figure 7: Headwind vs. Xwind

ROT: Crosswind Component

Degrees off nose	30°	45°	60°
Xwind component	50%	70%	90%

Let's look at non-right triangles. What if the crosswind is not 90-degrees off our course? We need a rule of thumb for headwind component as well. If we take the sine of three recognizable angles we can memorize only three multipliers for both headwind and crosswind estimates. As seen in **Figure 7** to the left, a 20 kt wind that is 30-degrees off the nose would only count as 10 kts of crosswind – or 50%. 20 kts at 45-degrees would count as 14 kts of crosswind – or 70%. 20 kts at 60-degrees would be 17.32 kts. For ease of handling in our Rule Of Thumb, we will round that up to 18 kts, 90% of the actual wind value. For more than 60-degrees off the nose, count the crosswind value as 100%. Remember, wind speeds and directions are immediately variable.

Aviation Math Job Performance Aids:

These mathematical and trigonometric concepts are incorporated into several electronic and mechanical aviation calculators. The Rules Of Thumb (ROT) that are offered here are intended to provide Ball Park Estimates (BPE) for use in flight.

For answers to FAA computer tests, or ground school knowledge tests, please use an electronic calculator. Carry the decimal point to its full extent until the very last minute - then round off to the required accuracy for the answer.

THAT'S IT!

So there you have it. Mathematical concepts play a major part in aviation. The better you understand these concepts, the more creative you can be. Use them to find other relationships.

But remember, even if you are a math whiz,

DON'T DO MATH IN THE COCKPIT!

MATH ROT FOR PILOTS
(Rules of Thumb):

1- DON'T DO MATH IN THE COCKPIT.

2- Round-off for the knowledge test after all calculation
Round-off in the real world with an axe.

3- TAS ≈ IAS + 2% per 1000 ft.

4- Xwind of 10% of TAS ≈ 6° of drift.

5- RWY required: Worst case + comfort factor.

6- 60:1 - One-degree at 60 miles ≈ one mile wide.

7- Glideslope VSI ≈ 1/2 GS x 10 (100 kts ≈ 500 fmp).

8- One-degree of pitch ≈ 100 fpm.

9- Square root of the Load Factor: For percentage of max gross:

Load		Square Root
60%	≈	0.75
70%	≈	0.85
80%	≈	0.90
90%	≈	0.95

10-Crosswind Component:

Degrees off nose	30°	45°	60°
Xwind component ≈	50%	70%	90%

PARTIAL CONVERSION TABLE

Length
1 in = 2.54 cm
1 m = 39.37 in
1 m = 3.281 ft
1 km = 0.621 mi
1 km = 0.540 nm
1 mi = 0.869 nm
1 mi = 1.609 km
1 nm = 1.151 mi
1 nm = 1.852 km

Mass
1 kg = 2.205 lb
1 lb = 0.454 kg

Temperature
F = (C x 9/5) +32
C = (F-32) x 5/9
K = C+273.15

Energy
1 J = 0.738 ft lbs
1 cal = 4.186 J
1 Btu = 252 cal

Power
1 HP = 550 ft lb /sec
1 HP = 0.746 kW
1 W = 1 J/sec

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